Application of the Goore Scheme to Turbulence Control for Drag Reduction (Ⅱ)
- Application to Turbulence Control -

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In Part Ⅰ, we extended the capability of the Goore Scheme for application to multi-dimensional problems and improved convergence performance. In this paper, we apply the improved Goore Scheme to the control of turbulence for drag reduction. Direct numerical simulations combined with the control scheme are carried out to simulate a controlled turbulent channel flow at low Reynolds number. The wall blowing and suction is applied through the Goore algorithm using the total drag as feedback. An optimum distribution of the wall blowing and suction in terms of the wall-shear stresses in the spanwise and streamwise directions is sought. The best case reduces drag by more than 20 %.

Key Words: Turbulence Control, Goore Scheme, Direct Numerical Simulation

1. Introduction

For the last decade, many researchers have searched for efficient control schemes which can reduce skin-friction drag on a flat plate. Most of them carried out numerical investigations since it is much easier to test any control scheme in numerical simulation than in experiment. The most widely selected control input so far is distributed wall blowing and suction at the wall (Choi *et al* 1994, Bewley and Moin 1995, Lee *et al*, 1997, Lee *et al*. 1998, Koumoutsakos 1999). Their schemes explicitly try to find an optimum distribution of blowing and suction that can reduce a specific cost function that is strongly correlated with drag. The optimal/suboptimal control theory, or nonlinear neural networks as well as physical intuitions have been tested to find it. Some of them successfully found the optimum wall blowing and suction. The performance, however, greatly depends on the selected parameter such as the cost function that is to be minimized in an optimal or suboptimal application. For example, when squared spanwise shear stress integrated over the wall is selected to be minimized, the scheme successfully reduces drag and the optimum wall actuation is represented as a function of the spanwise shear. When the total drag is selected instead, then a suboptimal approach fails to provide the optimum actuation, hence requiring an optimal approach which is totally unrealistic since it requires integrating the corresponding adjoint equation backward in time using the full information inside the flow domain. When a neural network is constructed to find the best correlation between the control input and wall-measurable quantities, then the control law results in a limited form such as the desired control input as a function of the wall-shear stress which was used in the training of the network. Therefore, one should be careful in the construc-

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tion of the control scheme. It is difficult to expect that one successful control scheme found in one formulation can be applied equally to other problems. This trend of problem-dependent behavior is mainly due to the nonlinear nature of turbulence.

The Goore Scheme is appropriate where the explicit relation between the control input and output is not known *a priori*. Furthermore, there is no restriction in selecting the control input and output. In Part I (Lee *et al*., 2001), we examined the original Goore Scheme and investigated its performance. We extended the scheme for multi-dimensional applications and validated it mathematically. We also improved the performance of the scheme by introducing a dynamic memory variable. In the present paper, we apply the improved Goore Scheme to the control of a turbulent channel flow for drag reduction. We combine the scheme with a direct numerical simulation of turbulent channel flows.

The paper is organized as follows. The numerical method used in our direct numerical simulation of a turbulent channel flow is briefly explained in Sec. 2. In Sec. 3, we discuss our application of the Goore Scheme using the spanwise wall shear stress Sec. 4, we present control results obtained by using the streamwise shear stress. We conclude our study in Sec. 5.

2. Numerical Methods

A turbulent channel flow in a box of $(4\pi, 2, 4\pi/3)h$ (streamwise, wall-normal, spanwise) is chosen for all simulations where h is the channel half gap. Periodicity is imposed in the streamwise $(x-)$ and spanwise $(z-)$ directions while Chebyshev functions are used in the expansion in the wall-normal $(y-)$ direction. (u, v, w) are the velocity components in (x, y, z) directions, respectively. The Crank-Nicolson scheme is used for the integration of the viscous terms and the third order Runge-Kutta scheme is adopted for the nonlinear terms. Detailed algorithm can be found in Kim *et al*. (1987). The mass flow rate is maintained to be constant throughout all tests of control schemes and the mean-wall skin friction in the streamwise direction is monitored for the performance of controls. The Reynolds number based on the wall-shear velocity u_τ, h and viscosity ν is 100. Control input is the wall blowing and suction distributed over both walls. The average of the mean skin frictions at the two walls is used for feedback at each time step. In order to apply the multi-dimensional version of the Goore Scheme, the number of degrees of freedom in the control parameter is maintained below 5 as suggested by Lee *et al*. (2001), which directs us to choose a control law as a function of local distribution of the wall-shear stress. We tested the streamwise and spanwise wall-shear stress independently. Therefore, we seek simple control laws between the wall shear stress and the wall blowing and suction based on the behavior of the mean skin friction. There is no delay between sensing and actuation, the control is applied at every time step.

3. Control Using the Spanwise Shear Stress

Lee *et al*. (1997) have shown that the spanwise shear stress at the wall can serve as a good indicator of the near-wall streamwise vortices rather than the streamwise shear stress. Thus we test first the wall blowing and suction as a function of the local spanwise shear stress. Assuming linearity yields

$$\phi_{ij}=C\sum_{i'=-I}^{I}\sum_{j'=-J}^{J} W_{i'j'}\frac{\partial w}{\partial y}\bigg|_{i+i',j+j'} \qquad (1)$$

where ϕ_{ij} is the wall blowing and suction and $\frac{\partial w}{\partial y}\big|_{ij}$ denotes the wall-shear stress in the spanwise direction with i and j denoting the grid indices in the streamwise and spanwise directions, respectively. The grid sizes are $\Delta x^+=40$, $\Delta z^+=13$ and $max(\Delta y^+)=6$. C has a dynamic property and is determined to yield a prescribed value of the root-mean-squared value of the wall blowing and suction, $0.15u_\tau$. Without any loss of generality, the absolute value of W_{ij} is bounded by 1. When symmetry is considered, the number of the independent weights can be reduced. Under the coordinate transformation, $z \to -z$ with $w \to -w$, the control law should be invariant. This implies that the weight is an odd function in the spanwise coordinate, $W_{i,-j} = -W_{ij}$. Therefore, $W_{i0}=0$ for all i's. By a similar argument, it can be shown that the weight is an even function in the streamwise coordinate, $W_{i,j}=W_{-i,j}$. With the reduced number of weights, Eq. (1) can be written as

$$\phi_{ij} = C\Big(\sum_{J=1}^{J} W_{0J}\Big(\frac{\partial w}{\partial y}\Big|_{i,j+J} - \frac{\partial w}{\partial y}\Big|_{i,j-J}\Big) + \sum_{I=1}^{I'}\sum_{J=1}^{J'} W_{IJ}\Big(\frac{\partial w}{\partial y}\Big|_{i+I,j+J} + \frac{\partial w}{\partial y}\Big|_{i-I,j+J} + \frac{\partial w}{\partial y}\Big|_{i+I,j-J} + \frac{\partial w}{\partial y}\Big|_{i-I,j-J}\Big)\Big) \quad (2)$$

We first select $J=3$, $I'=J'=0$. Hence W_{01}, W_{02} and W_{03} are only independent weights to be determined through the use of the Goore Scheme. Also, without a loss of generality, W_{01} can be assumed to be 1 or -1. W_{02} and W_{03} are allowed to vary between -1 and 1. This is based on the assumption that the weight distribution has a compact support. Otherwise, the control law would not be useful. The resolution of the weights is 0.1, hence 21 possible values for each weight ranging from -1 to 1 are allowed.

Next, we choose a reward probability function as a function of the mean drag, $r(D)$. The necessary condition for the distribution of $r(D)$ is that $r(D_0)=0.5$ and $\frac{dr}{dD} \le 0$ where D_0 is the mean drag of the flow without control. This distribution implies that if the mean drag is increased/decreased, such input actuation is discouraged/encouraged with the reward probability less/greater than 0.5. Figure 1 shows the reward distribution used throughout our tests. Also, the dynamic memory is a direct function of drag, $n(D)$ without resorting to the reward probability. Three different memory functions are tested:

$$(i) \quad n=20 \quad (3)$$

$$(ii) \quad n=5+5\left(\frac{160-D}{35}\right)^2 \quad (4)$$

$$(iii) \quad n=3+7\left(\frac{160-D}{35}\right)^2 \quad (5)$$

and are shown in Fig. 2. For the static memory (i) and dynamic memor cases (ii, iii), the Goore Scheme is applied to control with a turbulent channel flow, and the result is shown in Fig. 3 in terms of the mean drag. With the static memory, the scheme works intermittently which is a typical behavior of the original Goore Scheme. The scheme has so large a memory value even at an off-optimum point that it takes too much time to get away from that point. With dynamic memory, however, the drag is stably long reduced by 20 %. When the system visits an off-optimum point, the scheme quickly escapes from there owing to the small size of memory while near the optimum point, the scheme hardly takes an excursion from it. Furthermore, the initial behavior in the transition period with dynamic memory shows a much

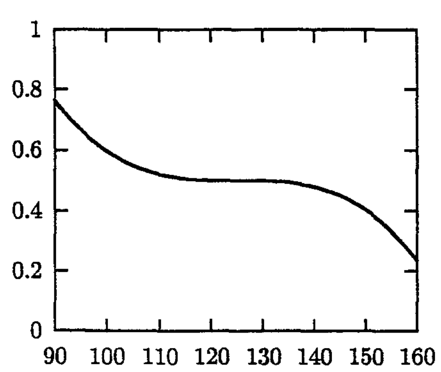


Fig. 1 Reward function as a function of drag

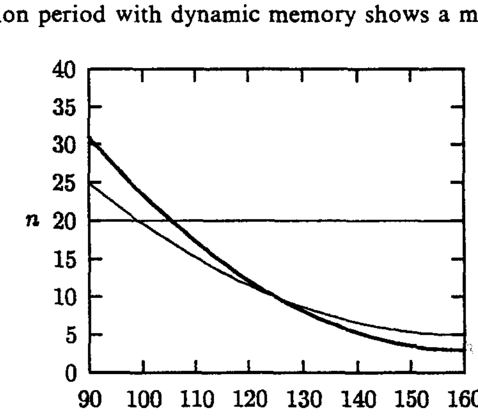


Fig. 2 Dynamic memory functions as functions of drag: dashed line, $n=20$; thin solid line, $n=5+5\left(\frac{160-D}{35}\right)^2$; thick solid line, $n=3+7\left(\frac{160-D}{35}\right)^2$

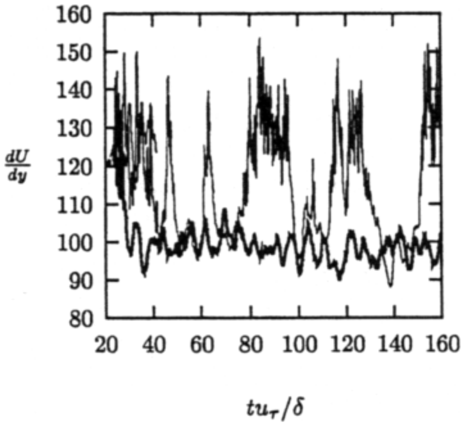
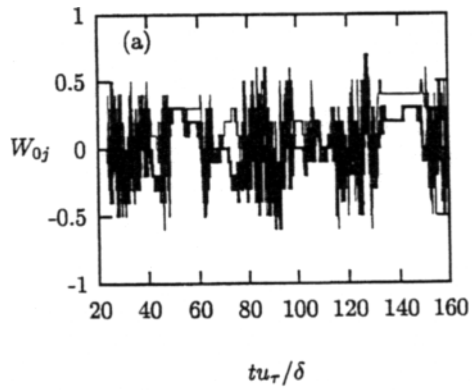


Fig. 3 Mean wall-shear for controls using the spanwise wall-shear stress; dashed line, no-control case; thin solid line, constant memory case; thick solid line, dynamic memory function case

faster convergence than that with static memory.

For these two cases, the convergence performance of the weights is shown in Fig. 4. For both cases, W_{01} remains at 1 for more than 90 percent of the time (figure not shown here). Compared to the static memory case where convergence is very chaotic, the dynamic memory case shows an excellent convergence behavior. The converged weight values are W_{02}=0.1 and W_{03}=0.2. These values are different from the values reported in the previous study using neural networks with a similar control law (Lee *et al.* 1997) where W_{02}= 0.0 and W_{03}=0.33. Considering that both controls reduce drag by 20 %, an optimum weight distribution seems to be not unique. It should be noted that the control using a neural network tries to minimize a specific object function while our control using the Goore Scheme searches an actuation directly to reduce drag. Here, we again place an emphasis on the fact that the Goore Scheme is supposed to find the optimum input such that any object function be minimized without any specific knowledge on the relation between the input and the object function. We also need to mention that by employing dynamic memory, to some extent we weaken the capability of the original scheme to find a *global* optimum point in the system state space. Our modified scheme tends to stick to a *local* optimum point



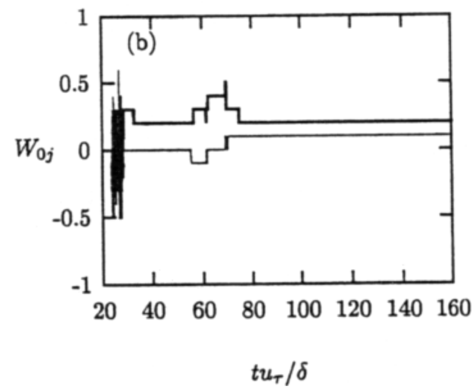


Fig. 4 Convergence of the weights for the controls using the spanwise wall-shear stress; (a) constant memory case; (b) dynamic memory function case. Thin solid line denotes W_{02} and thick solid line denotes W_{03}

once it finds one. However, if we run the scheme long enough, the scheme is supposed to find a *global* optimum since the scheme provides every point a fair chance to become an optimum point in the long run.

The probability density functions of the weights are shown in Fig. 5 for both static and dynamic memory cases. The broken line represents the binomial distribution which is a distribution the system would produce if there were no preferred distribution. The dynamic memory case shows pronounced peaks while the constant memory case does not. It is interesting to note that although pronounced peaks do not exist for the constant memory case, the scheme still reduces drag intermittently. We may need a conditional pdf to capture the pattern in the weight distribution.

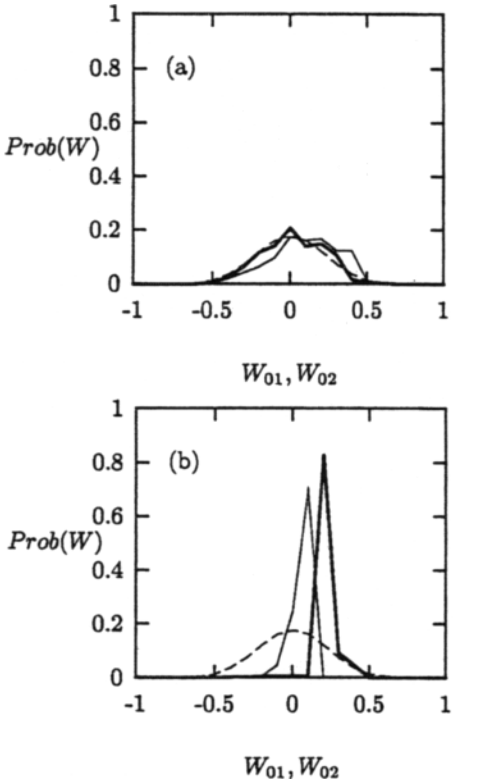


Fig. 5 Probability density function of the weights for the controls using the spanwise wall-shear stress: (a) constant memory case; (b) dynamics memory function case. Thin solid line denotes $Prob(W_{01})$ and thick solid line denotes $Prob(W_{02})$

We carried out a series of numerical test with different template sizes. An increase in J did not improve the control performance and the extra information along the streamwise direction did not either. From this we conclude that $J=3$ is the optimum, suggesting that the spanwise information along 90 wall units is enough to determine actuation at one point.

4. Control Using the Streamwise Shear Stress

Among the components of the wall-shear stress, the streamwise one is the first natural choice for control parameter since it directly relates to the mean shear and is easily measurable in real practice by using a recent MEMS (Micro-Electromechanical Systems) technology (Tsao *et al.*, 1999). A previous control scheme (Lee *et al.* 1998), however, has shown that in a suboptimal control approach a selection of the streamwise shear stress as a control parameter is not so successful. The streamwise shear stress does not play as a good indicator of the near-wall streamwise vortices reflected onto the wall as the spanwise shear stress does. In this section, we investigate if there exists any successful control law in terms of the streamwise shear stress by using the Goore Scheme because the scheme is ideal for finding any clue leading to a successful control scheme.

The following control formula is assumed:

$$\phi_{ij}=C\sum_{\ell=-I}^{I}\sum_{\jmath=-J}^{J} W_{\ell\jmath}\frac{\partial u}{\partial y}\bigg|_{i+\ell,j+\jmath} \qquad (6)$$

where $\frac{\partial u}{\partial y}$ denotes the fluctuation component of the streamwise shear stress. By considering the fluctuation component, the restriction, $\Sigma_{\ell=-I}^{I}\Sigma_{\jmath=-J}^{J}\ W_{\ell\jmath}=0$ is eliminated. The spanwise symmetry holds, *i.e.*, the control law should be invariant under $z\rightarrow -z$. Thus, $W_{i,j}=W_{i,-j}$. However, the streamwise symmetry is not valid because of the upstream-downstream asymmetry. We tested even and odd modes in the streamwise direction independentely. For the odd modes, $W_{i,j}=-W_{-i,j}$ with $W_{0j}=0$. The control law then becomes

$$\phi_{ij}=C\Bigg(\sum_{\ell=1}^{I} W_{\ell 0}\bigg(\frac{\partial u}{\partial y}\bigg|_{i+\ell,j}-\frac{\partial u}{\partial y}\bigg|_{i-\ell,j}\bigg)$$
$$+\sum_{\ell=1}^{I}\sum_{\jmath=1}^{J} W_{\ell\jmath}\bigg(\frac{\partial u}{\partial y}\bigg|_{i+\ell,j+\jmath}-\frac{\partial u}{\partial y}\bigg|_{i-\ell,j+\jmath}+\frac{\partial u}{\partial y}\bigg|_{i+\ell,j-\jmath}-\frac{\partial u}{\partial y}\bigg|_{i-\ell,j-\jmath}\bigg)\Bigg) \qquad (7)$$

The control law for the even modes with $W_{i,j}=W_{-i,j}$ becomes

$$\phi_{ij}=C\Bigg(W_{00}\frac{\partial u}{\partial y}\bigg|_{i,j}+\sum_{\ell=1}^{I} W_{\ell 0}\bigg(\frac{\partial u}{\partial y}\bigg|_{i+\ell,j}+\frac{\partial u}{\partial y}\bigg|_{i-\ell,j}\bigg)+\sum_{\jmath=1}^{J} W_{0\jmath}\bigg(\frac{\partial u}{\partial y}\bigg|_{i,j+\jmath}+\frac{\partial u}{\partial y}\bigg|_{i,j+\jmath}\bigg)$$
$$+\sum_{\ell=1}^{I}\sum_{\jmath=1}^{J} W_{\ell\jmath}\bigg(\frac{\partial u}{\partial y}\bigg|_{i+\ell,j+\jmath}+\frac{\partial u}{\partial y}\bigg|_{i-\ell,j+\jmath}+\frac{\partial u}{\partial y}\bigg|_{i+\ell,j-\jmath}+\frac{\partial u}{\partial y}\bigg|_{i-\ell,j-\jmath}\bigg)\Bigg) \qquad (8)$$

The templates for both modes are shown in Fig. 6.

Both controls are tested in the same turbulent channel flow. Mean shear distributions for both schemes are shown in Figs. 7 and 8, respectively. The odd-mode control does not reduce drag at all

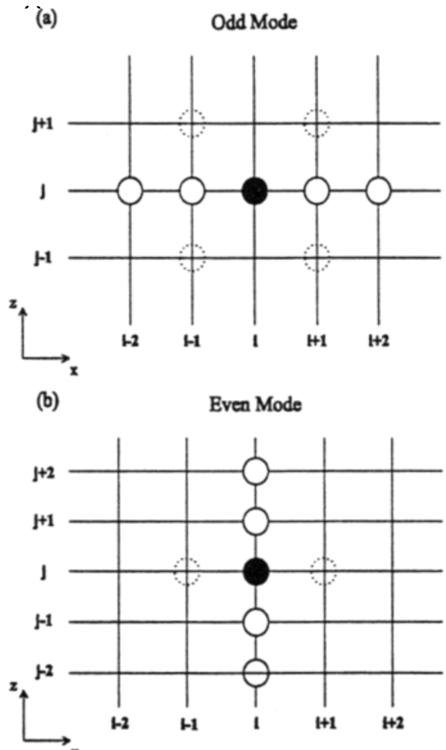


Fig. 6 Weight locations for (a) the odd mode and (b) the even mode

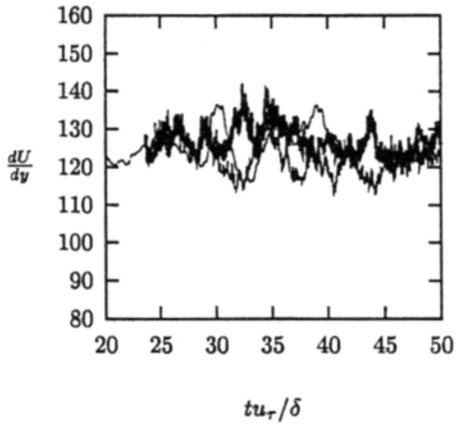


Fig. 7 Mean wall-shear for the controls using the streamwise wall-shear stress (odd mode): dashed line, no-control case; thin solid line, $I=2$, $I'=J'=0$; thick solid line, $I=2$, $I'=J'=1$

while the even-mode control reduces drag by 12 %. The odd-mode control is mathematically similar to $\phi \sim \frac{\partial}{\partial x}\frac{\partial u}{\partial y}\Big|_{wall}$. The suboptimal control minimizing squared streamwise shear stress integrated over the wall produced a similar control law and did not reduce drag (Lee *et al.* 1998). The even-mode control scheme roughly represents $\phi \sim \frac{\partial u}{\partial y}\Big|_{wall}$, physically meaning that blowing/suction is applied to high/low skin-friction region. The corresponding probability distributions for W_{01} and W_{02} are shown in Fig. 9. Peaks are not well pronounced for W_{02} while the distribution of W_{01} clearly shows a shift of the peak in the positive direction. We also tested the even-mode control by adding more points in the streamwise direction as well as in the spanwise direction, resulting in no improvement. Although our result supports the intuition-based control that blowing/suction is applied to high/low skin-friction area, two comments should be made. First, when such an intuition-based control, $\phi = C\frac{\partial u}{\partial y}\Big|_{wall}$, was actually applied, the scheme did not reduce drag. The reason is that the streamwise wall-shear stress is so sensitive to blowing and suction, and once control is applied, the shear stress does not contain proper information about the near-wall structure. However, our control using the even-mode does not experience such a difficulty, meaning that the Goore Scheme possesses the capability of relaxing the sensitivity involved in the control. Second, although the probability distribution of the weight shows a pronounced peak at a non-zero value, in fact the weight constantly changes with time. This dynamic trend of the weight appears to play a essential role in making the scheme work. Hill (1993) derived a simple control law based on the streamwise wall-shear stress by a local optimization that reduces drag by an amount similar to ours. However, their approach involves an approximation of the near-wall dynamics, thus cannot be applied to a different kind of problem in the same manner. Our control using the Goore Scheme, however, does not require such a problem-dependent formulation and can be easily applied to any dynamical system.

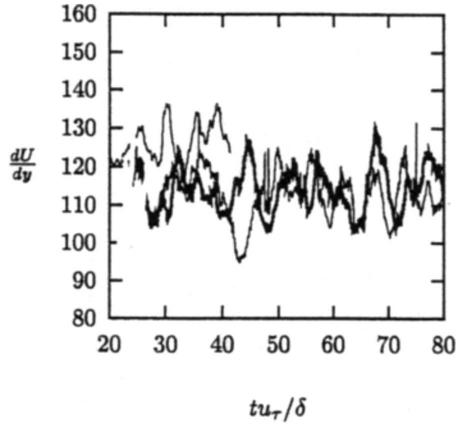


Fig. 8 Mean wall-shear for the controls using the streamwise wall-shear stress (even mode): dashed line, no-control case; thin solid line, $I=1$, $J=2$, $I'=J'=0$; thick solid line, $I=0$, $J=2$, $I'=J'=1$.

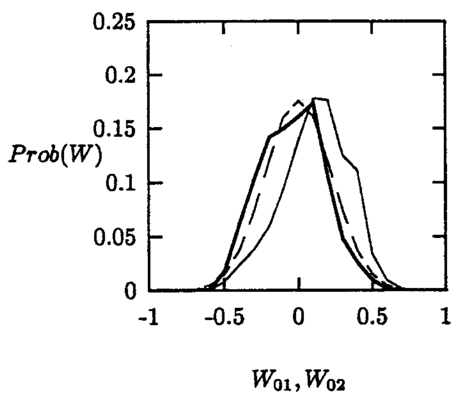


Fig. 9 Probability density function of the weights for the controls using the streamwise wall-shear stress (even mode); dashed line, binomial distribution; thin solid line, W_{01}; thick solid line, W_{02}

5. Conclusion

We have tested the improved Goore Scheme in the control of turbulent channel flow for drag reduction. Dynamic memory clearly improve the control performance. Wall blowing and suction are applied as a weighted sum of the local shear stress at the wall. The Goore Scheme finds an optimum distribution of these weights based on the response of the total drag to the control input. When the spanwise shear stress is adopted, the scheme reduces drag by 20 %. It also provides a pattern in the distribution of the weights which is similar to the one found in the control scheme using neural network. When the streamwise shear stress is used, drag is reduced only by 12 %. A fixed pattern is not observed with the streamwise shear stress. We have not carried out an extensive parametric study such as testing the scheme at different Reynolds numbers. What we focus on in this study is feasibility of the Goore Scheme in the control of a very complicated nonlinear dynamical system. The application of the Goore Scheme in our study is so straightforward that it can be easily applied to any other dynamical system with an unknown mechanism.

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